The problem is formulated with 48 decision variables

DECISION VARIABLES:

x1_1, x1_2, x1_3, x1_4, x2_1, x2_2, x2_3................x12_1, x12_2, x12_3, x12_4

of the form x<month_number>_<crop_number>

OBJECTIVE FUNCTION:

sum of([price_per_ton(j)*yield_per_hectare(j)- cost_per_hectare(j)]*xi_j)

for month i and crop j

INEQUALITY CONSTRAINTS:

Example: 5.16*x1_3 + 0.75*x1_4 <= 31 (number of days in january)

A is coefficient matrix of Inequality constraints

EQUALITY CONSTRAINTS:

Example: x1_1 = 0, x1_2 = 0 etc (rice in jan is zero, maize in jan is zero

Lower limit = 0

*In the MATLAB code, the objective function, inequality and equality constraints are defined as matrices **f, A, B, Aeq and Beq** which are coefficient matrices of decision variables and using them as inputs for the function **LINPROG**.*

Each row in A and Aeq matrices represents ONE constraint. Since there are 48 decision variables, the number of columns in both A and Aeq is 48. Vectors B and Beq represent the RHS of each constraint.

Each element of vector f represents coefficient of decision variable in objective function.

LINPROG solver
[x,fval]=linprog(f,A,b,Aeq,beq,lb,ub)
this function solves the following linear programming problem
min f*x
subject to
    A*x <=b
    Aeq * x = beq
    lb <= x <= ub
the return variables are :
x : the solution
fval: value of the objective function at the optimal solution

FORMULATION

$$\max \quad \sum_{i=1}^{12}\sum_{j=1}^{4}(p_j y_j - c_j)x_{ij} \; - \; h\left(\sum_{j=1}^{4} M_{ij}x_{ij} \; - 25\right) + \; f\left(25 - \sum_{j=1}^{4} M_{ij}x_{ij}\right)$$

$$subject\ to \qquad \sum_{j=1}^{4} M_{ij}x_{ij} \;\le\; N_i \qquad \forall\, i = 1,2, \ldots\ldots,12$$

$$\sum_{j=1}^{4} x_{ij} \;\le\; L \qquad \forall\, i = 1,2, \ldots\ldots,12$$

$$x_{ij} \;=\; 0 \qquad for\ specific\ crop\ j\ and\ month\ i$$

$$x_{ij} \ge\; 0 \qquad \forall\;\, i,j$$

Where

x_{ij}: DECISION VARIABLES hectares of land used for crop *j* in month *i*

p_j: price per ton of crop *j*

y_j: yield per hectare of crop *j*

c_j: cost per hectare of crop *j*

h: cost of hiring a day's labor

f: revenue from family doing off-field labor

M_{ij}: man days per hectare required for crop *j* in month *i*

L: available land in every month

The formulated problem is a Linear Programming Problem with a conditional objective function. Where the last two terms of the objective function are depending on the condition whether the man days of labor is less than 25 days or not.

To reduce the complexity while solving the problem in MATLAB, an assumption was made that the man days required every month by the optimal solution and its effect on the objective function will not affect much greatly. Therefore in MATLAB, only the first term of objective function is taken into account and the total revenue is calculated at last by adding the last two terms to the optimal objective function value.

After taking the assumption into account, the problem becomes purely Linear with linear equality and inequality constraints. Hence, **linprog** solver is used in MATLAB.

RESULTS OF THE MATLAB OPTIMIZATION

x =

1	0	**11**	1.67E-08	**21**	4	**31**	0	**41**	2.5
2	0	**12**	8.00E-09	**22**	6.20E-09	**32**	0	**42**	0
3	4	**13**	4	**23**	0	**33**	4	**43**	2.09E-08
4	1.43E-08	**14**	0	**24**	0	**34**	6.20E-09	**44**	2.27E-08
5	0	**15**	1.59E-08	**25**	4	**35**	0	**45**	0
6	0	**16**	7.80E-09	**26**	6.30E-09	**36**	0	**46**	0
7	4	**17**	3.444444	**27**	0	**37**	1.065916	**47**	4
8	1.42E-08	**18**	2.24E-08	**28**	0	**38**	2.934084	**48**	1.43E-08
9	4	**19**	0	**29**	4	**39**	0		
10	0	**20**	0	**30**	6.30E-09	**40**	0		

number_of_days_used =

1	20.64	**1**	6
2	20	**2**	8
3	20	**3**	4
4	20	**4**	31
5	31	**5**	30
6	8	6	18.72

number_hectacres_used =

1	4	1	4
2	4	2	4
3	4	3	4
4	4	4	4
5	3.444	5	2.5
6	4	6	4

Total_Revenue =

18114.552

The results found are subject to the assumptions stated above in the report. Also there is one more assumption that every month, after the end of the month the solution says that another different crop takes its place instantly. It is a bit absurd and data such as time taken to clear the field and time taken to make it ready for next crop cultivation is required.

Overall, the solution found is highly theoretical and will be proven wrong in any practical cases. It clearly cannot be used a base for real-world scenarios.